Arc segmentation in linear time

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Contribution

We propose a linear algorithm based on discrete geometry approach for segmentation of a curve into digital arcs.

Motivation

– Arc and circle are basic object in discrete geometry.
– Arc and circle appear often in images.
– Shape contains often digital arcs.
⇒ The study of these primitives is important.

Discrete circle

– Basic object in discrete geometry.
– Based on discretization of a real circle.

Discrete line [Reveillès91]

A discrete line, noted as \( D(a, b, \mu, \omega) \), \( a, b, \mu, \omega \in \mathbb{Z}^2 \), is a set of points that verifies: \( \mu \leq ax - by \leq \mu + \omega \).

Blurred segment [Debled et al. 06]

A blurred segment \( \nu \) is a set of points that satisfies:

– There exist a discrete line \( D(a, b, \nu, \omega) \) that contains this set.

\( \max(|\nu_x|, |\nu_y|) \leq \nu \)

Tagent space [Latecki00]

Input:
Suppose that \( C = \{C_i\}_{i=0}^{n} \) is a polygonal curve
– \( \alpha_i = \angle(C_{i-1}C_iC_{i+1}) \)
– \( l_i \) the length of segment \( C_iC_{i+1} \).
– \( \alpha_i > 0 \) if \( C_{i+1} \) is on the right side of \( C_{i-1}C_i, \alpha_i < 0 \) otherwise.

Output:
We consider the transformation that associates \( C \) to a polygon of \( \mathbb{R}^2 \) constituted by segments \( T_\alpha T_{(i+1)} T_{(i+1)} T_{(i+1)2} \) for \( i \) from 0 to \( n-1 \) with

– \( T_{i0} = (0, 0) \)
– \( T_{i1} = (T_{i-1,2} x + l_{i-1}, T_{i-1,2} y), 1 \leq i \leq n \)
– \( T_{i2} = (T_{i,1} x + T_{i,1} y + \alpha_i), 1 \leq i \leq n-1 \).

Deciding if a curve is an arc

1. Polygonalize the input digital curve by polygon \( P \) based on recognition of BS of width 1.
2. Transform \( P \) to tangent space \( T(P) \) in the tangent space.
3. Determine the midpoint set \( MP C = \{M_i\}_{i=1}^{n+1} \) of horizontal segment of \( T(P) \).
4. Verify if \( MP C \) is a BS of width \( \epsilon \) [Debled 06]

Interest of proposition 1

Arc detection → Recognition of discrete line

Example:

Experimental results:

Study of quasi co-linear property

Convergence of radius of local circumcircles

Proposition 2:
Let \( C = \{C_i\}_{i=0}^{n} \) be a polygon, \( \alpha_i = \angle(C_{i-1}C_iC_{i+1}) \). The length of \( C_iC_{i+1} \) is \( l_i \), for \( i \in \{0, \ldots, n-1\} \). We denote \( O_j, R_j, H_j \) respectively the center and the radius of circumcircle that passes to 3 points \( C_{i-1}, C_i, C_{i+1} \), the projection of \( O_i \) on \( C_iC_{i+1} \), suppose that \( R_i - \delta H_i \leq h \) for \( i \in \{1, \ldots, n-1\} \). This results below is obtained.

\[ R_i h \leq \frac{H_i}{\alpha_i} \leq R_i (\alpha_i - 0.33 \pi h) \].

Convergence of centers of local circumcircles

Proposition 2:
Let us consider a sequence of points \( \{C_i\}_{i=0}^{n} \). We denote \( O'_i \) (resp. \( O_i \) ) and \( R'_i \) (resp. \( R_i \) ) are the center and radius of circumcircle that passes to 3 points \( C_0 \) (resp. \( C_i \)), \( C_i, C_{i+1} \). There exist \( R, \delta \) such that \( R, \delta \leq \frac{R_i - \delta H_i}{2} \), for \( i = 1, \ldots, n-1 \). Suppose that \( \angle C_iC_{i+1}C_{i+2} > \frac{\pi}{2} \) for \( k \in \{0,1\}, k < n \). Therefore, we have this property \( 0 \leq R'_i \leq R, \delta \leq \delta, 0 \leq H'_i \leq R_i - \delta \), for \( 1 \leq i \leq n-1 \).

Arc in the tangent space

Proposition 1:
Let \( C = \{C_i\}_{i=0}^{n} \) be a polygon, \( \alpha_i = \angle(C_{i-1}C_iC_{i+1}) \). The length of \( C_iC_{i+1} \) is \( l_i \), for \( i \in \{0, \ldots, n-1\} \). The vertices of \( C \) are on a real arc of radius \( R \) with center \( O \), \( \angle COC_{i+1} \leq \frac{\pi}{2} \) for \( i \in \{1, \ldots, n-1\} \). This results below is obtained.

\[ 1 \leq R \leq \frac{\alpha_i}{l_{i+1}} \leq \frac{1}{0.9742979 R} \]

Arc segmentation

Main ideas:
1. Polygonalize the input curve
2. Transform the polygon to tangent space
3. Construct the curve of midpoints in the tangent space
4. Polygonalize the midpoint curve
– Utilize parameter \( \alpha \) to verify detected arcs.