Image Denoising with a Constrained Discrete Total Variation Scale Space
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Contribution
We consider a combinatorial approach that relies on coupling the TV-flow (which corresponds to the solution of a differential inclusion) that incrementally simplifies the original noisy image with a procedure that intends to recover the contrast.

Notations
- Markovian framework:
  - set of pixels: \( V \)
  - value of image \( u \) at site \( i \): \( u_i \)
- set of interactions: \( W \)
- Discrete Total Variation (DTV)
  \[ J(u) = \sum_{(i,j) \in W} R_{i,j}(u) = \sum_{(i,j) \in W} |u_j - u_i| \]
- Sub-differential of \( F \) at \( x \)
  \[ \partial F(x) = \{ s | \forall y, \langle y - x, s \rangle + F(x) \leq F(y) \} \]
- Minimal subgradient of \( F \) at \( x \)
  \[ m(\partial F(x)) = \text{projection of } 0 \text{ onto } \partial F(x) \]

A coupled scale-space approach
Approach coupling two procedures
1. Procedure of simplification (denoising but loss of contrast) of the observed image: \( t \rightarrow u(t) \) solution of DTV-flow
2. Procedure that respects shapes and recovers the contrast: \( t \rightarrow \tilde{u}(t) \) is the image that is the closest to the observed image \( f \) having the same relative order as \( u(t) \). This corresponds to the projection of \( f \) onto the convex set:

\[ \bigcap_{(i,j) \in W} \{ g \in \mathbb{R}^N | |g_j - g_i| + m_i(\partial R_{i,j}(u(t)))(g_j - g_i) = 0 \} \]

Results
(a) Original image
(b) Noisy image
(a) Our result
(b) Residual
(a) TV minimizer
(b) Residual

Relative Order Preservation
- We want to keep the relative order of the level lines
- This constraint is maintained through:
  - constraining relative order between two interacting pixels
  - using Bregman distances
  \[ |u_j - u_i| + m_i(\partial R_{i,j}(u))(u_j - u_i) = 0 \]
  \( \Rightarrow \) Geometric information maintained as a variational form
- Need to select the minimal subgradient:
  - otherwise relative order not necessarily satisfied
  - required for convergence properties of the approach

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