Image Denoising with a Constrained Discrete Total Variation Scale Space

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Contribution

We consider a combinatorial approach that relies on coupling the TV-flow (which corresponds to the solution of a differential inclusion) that incrementally simplifies the original noisy image with a procedure that intends to recover the contrast.

Notations

- Markovian framework:
  - set of pixels : $\mathcal{V}$
  - value of image $\mathbf{u}$ at site i: u_i
  - set of interactions: $\mathcal{W}$
- Discrete Total Variation (DTV)

$$J(\mathbf{u}) = \sum_{(i,j)\in\mathcal{W}} R_{i,j}(\mathbf{u}) = \sum_{(i,j)\in\mathcal{W}} |u_j - u_i|$$

- Sub-differential of F at x

$$\partial F(\mathbf{x}) = \{\mathbf{s} \,|\, \forall \mathbf{y},\ \langle \mathbf{y}-\mathbf{x}, \mathbf{s}\rangle + F(\mathbf{x}) \leq F(\mathbf{y})\}$$

- Minimal subgradient of F at x

$$m(\partial F(x)) = \text{ projection of } 0 \text{ onto } \partial F(x)$$

Discrete Total Variation Flow

- DTV-flow (Differential Inclusion)

$$\begin{cases} \frac{d\mathbf{u}}{dt}(t) \in -\partial J(\mathbf{u}(t)) \text{ on } (0, +\infty) \\ \mathbf{u}(0) = \mathbf{f} \end{cases}$$

- The slow solution of the differential inclusion yields the trajectory of DTV-flow
- Computed exactly using a network-flow approach
- It generates a sequence of images that simplfies more and more the original image as time evolves
- It presents a **loss of contrast**

$\Rightarrow$ Idea: get back the constrast while preserving the geometric information

Relative Order Preservation

- We want to keep the relative order of the level lines
- This constraint is maintained through:
  - constraining relative order between two interacting pixels
  - using Bregman distances

$$|u_j - u_i| + m_i(\partial R_{i,j}(\mathbf{v}))(u_j - u_i) = 0$$

$\Rightarrow$ Geometric information maintained as a variational form

- Need to select the minimal subgradient:
  - otherwise relative order not necessarily satisfied
  - required for convergence properties of the approach

A coupled scale-space approach

Approach coupling two procedures

1. Procedure of simplification (denoising but loss of contrast) of the observed image: $t \mapsto \mathbf{u}(t)$ solution of DTV-flow
2. Procedure that respects shapes and recovers the contrast: $t \mapsto \tilde{\mathbf{u}}(t)$ is the image that is the closest to the observed image $\mathbf{f}$ having the same relative order as $u(t)$. This corresponds to the projection of $\mathbf{f}$ onto the convex set:

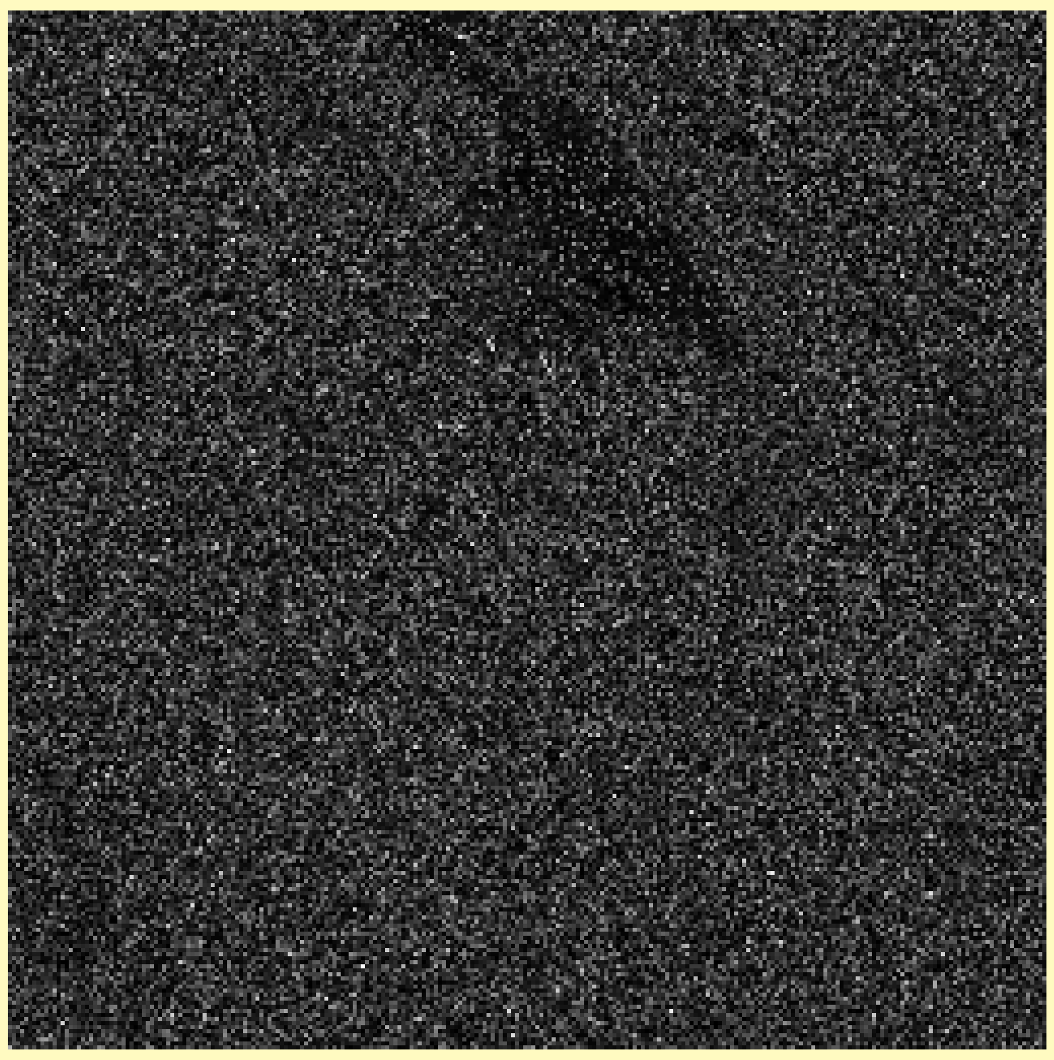
$$\bigcap_{(i,j)\in\mathcal{W}} \Big\{ \mathbf{g} \in \mathbb{R}^N \mid \underbrace{|g_j - g_i| + m_i(\partial R_{i,j}(\mathbf{u}(\mathbf{t})))(g_j - g_i)}_{\text{relative order for } (i,j)} = 0 \Big\}$$

Results

(a) Original image

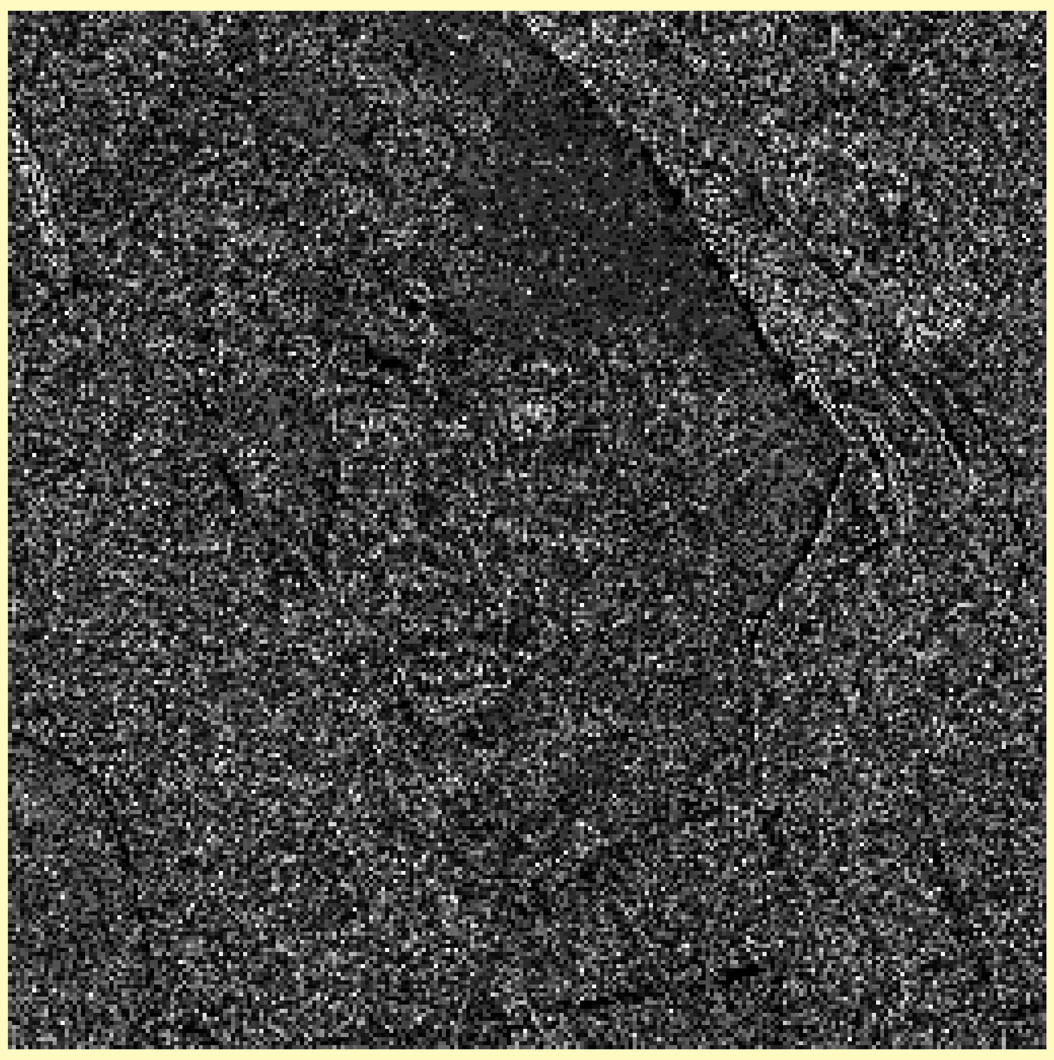
(b) Noisy image

(a) Our result



(b) Residual

(a) TV minimizer



(b) Residual

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