A near-linear time guaranteed algorithm for digital curve simplification under the Fréchet distance

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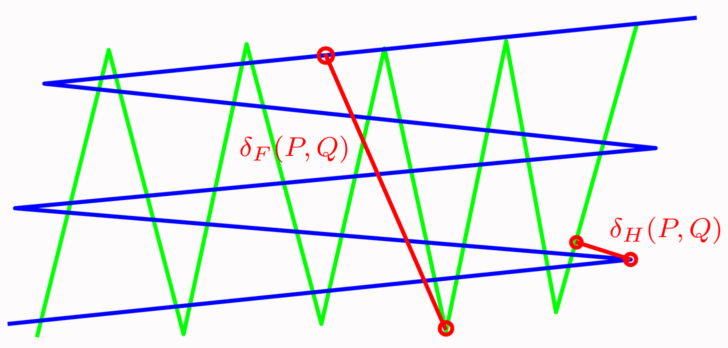
Grenoble
Images
Parole
Signal
Automatique

Fréchet distance

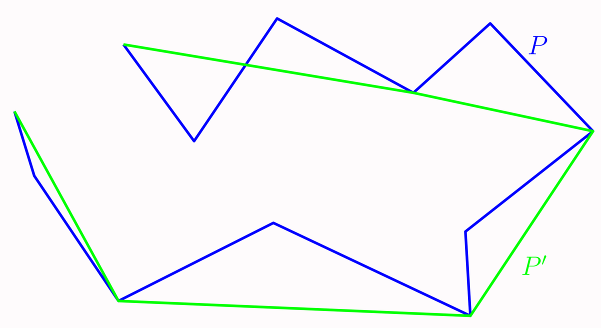


Hausdorff or $L_{\{1,2,\infty\}}$ distances are not good measures of the similarity of curves.
⇓
Fréchet distance takes into account the course of the curves.

Decide if $\delta_F(P,Q) \leq \varepsilon$ in $\mathcal{O}(mn)$.

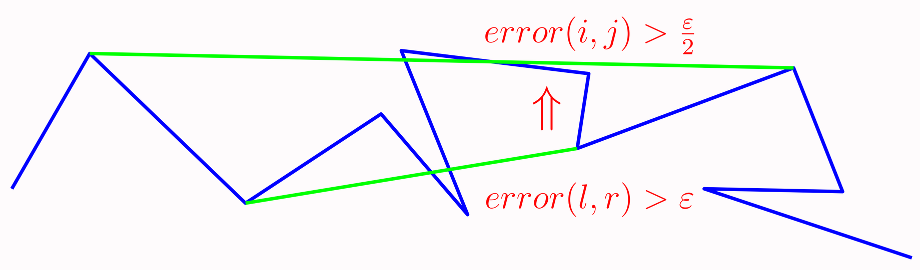
Curve simplification

Find P' an ε-simplification of P
=
Find shortcuts p_ip_j such that
$error(i,j) = \delta_F(p_ip_j, P) \leq \varepsilon$
+
minimize the number of vertices of P'



Optimal algorithm in $\mathcal{O}(n^3)$.

A nice local property [2]



Let $P = \{p_1, p_2, \ldots, p_n\}$ be a polygonal curve.
For $1 \leq i \leq l \leq r \leq j \leq n$, $error(l,r) \leq 2error(i,j)$.

Algorithm to compute an ε-simplification with at most the number of vertices of an optimal $\frac{\varepsilon}{2}$-simplification in $\mathcal{O}(n\log(n))$

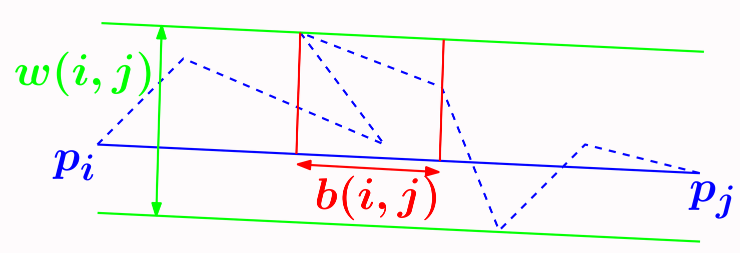
Guaranteed Algorithm with approximated distance

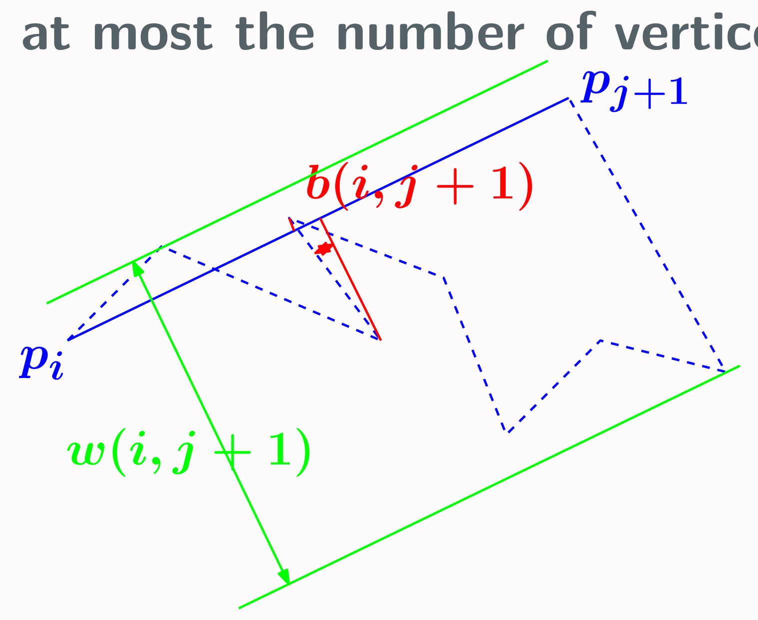
The Fréchet error of a shortcut p_ip_j satisfies [1] : $\max(\frac{w(i,j)}{2}, \frac{b(i,j)}{2}) \leq error(i,j) \leq 2\sqrt{2}\max(\frac{w(i,j)}{2}, \frac{b(i,j)}{2})$

Algorithme 1 : Greedy simplification algorithm
$i = 1$, $j = 2$
while $i < n$ **do**
  while $j < n$ *and* $\max(w(i,j), b(i,j)) \leq \frac{\varepsilon}{\sqrt{2}}$ **do**
    j=j+1
  create a new shortcut p_ip_{j-1}
  $i = j-1, j = i+1$

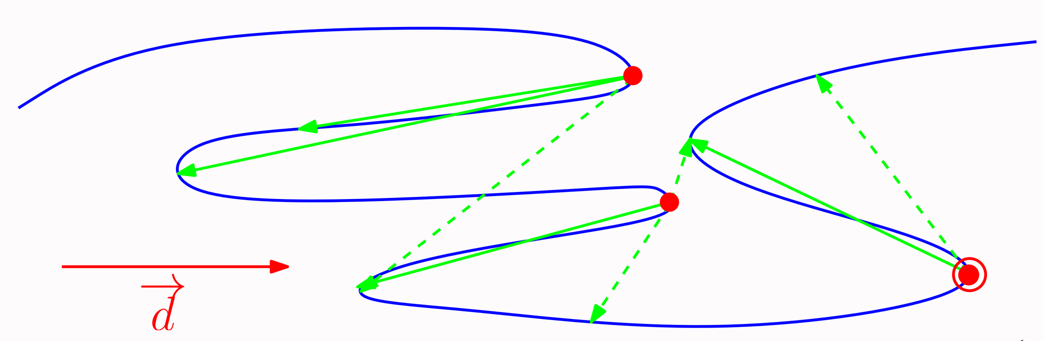
$\Longrightarrow$ **Computes an ε-simplification with at most the number of vertices of an optimal $\frac{\varepsilon}{4\sqrt{2}}$-simplification**

$\Longrightarrow$ **Complexity ? Efficient update ?**





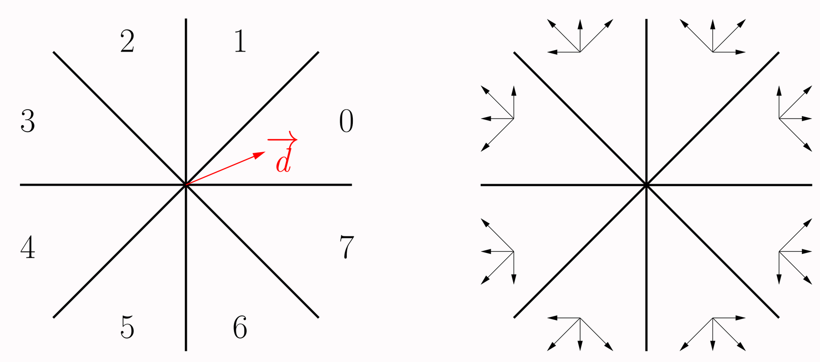
Approximated distance update



- occulter = locally furthest point in direction $\vec{d}$
- origin of the longest backpath = occulter

Keypoint : knowing when the curve goes forward or backward in a given direction

Digital curve : only 8 elementary shifts



An elementary shift	**always** goes forward **or** **always** goes backward

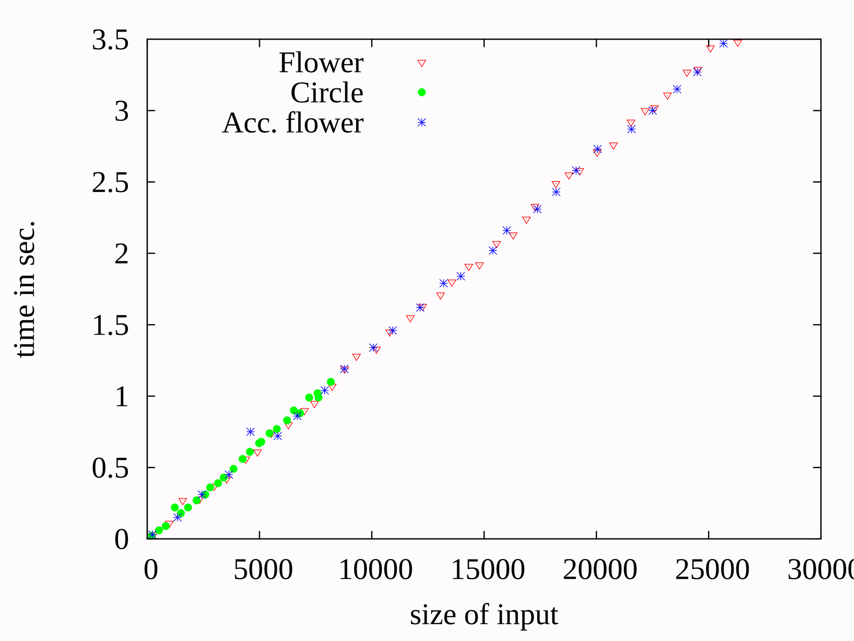
for **all the directions of an octant**.

- for all the directions of an octant, the occulters are the same
- ... but more than one active occulter per octant ! !

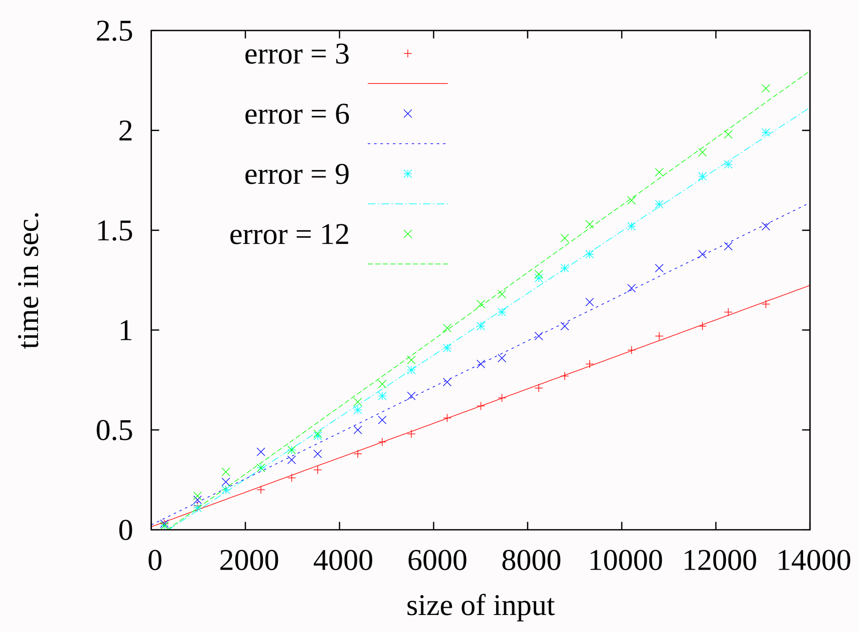
For digital curves, the number of active occulters per octant is bounded by ε.

Overall complexity $\mathcal{O}(n\log(n))$

Results

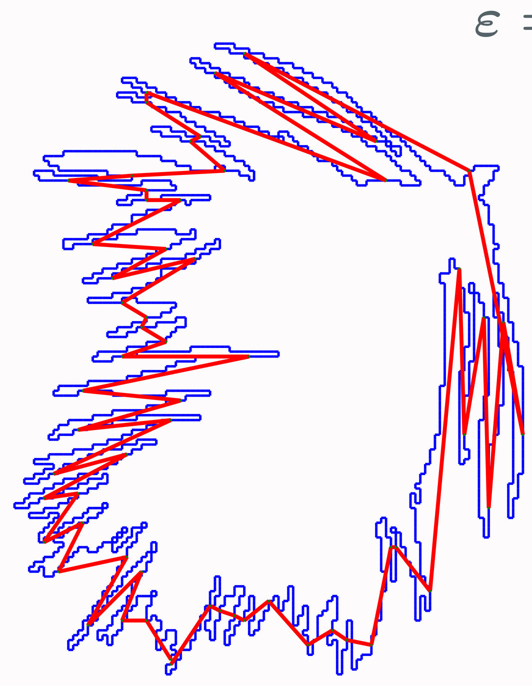


Runtime results for noisy synthetic shapes.

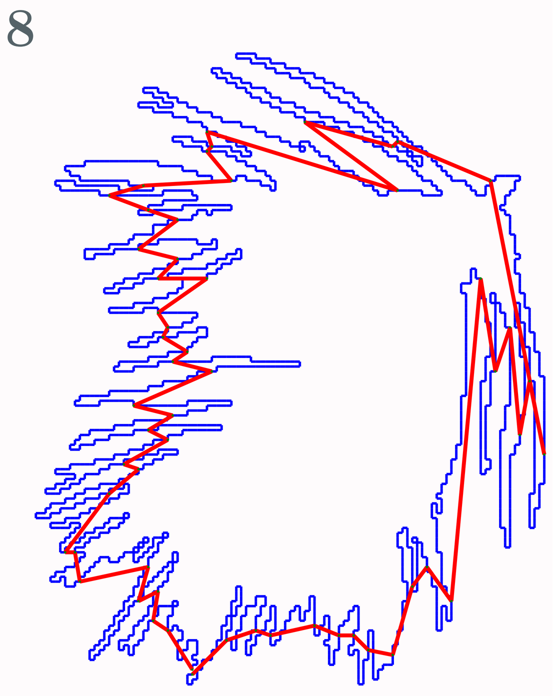


Runtime results for noisy flowers with different values for ε.

$\varepsilon = 8$



Approx. Fréchet distance
60 vertices



Width criterion
56 vertices

References

[1] Abam, M.A., de Berg, M., Hachenberger, P., Zarei, A. : Streaming algorithms for line simplification. In : SCG '07 : Symp. on Comput. geometry. pp. 175–183. ACM (2007)

[2] Agarwal, P.K., Har-Peled, S., Mustafa, N.H., Wang, Y. : Near-linear time approximation algorithms for curve simplification. Algorithmica 42(3-4), 203–219 (2005)

[3] Chan, W.S., Chin, F. : Approximation of polygonal curves with minimum number of line segments. In : ISAAC '92 : Symp. on Algorithms and Computation. pp. 378–387. Springer-Verlag (1992)