A near-linear time guaranteed algorithm for digital curve simplification under the Fréchet distance

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Fréchet distance

Hausdorff or $L_1(1,2,\infty)$ distances are not good measures of the similarity of curves.

Fréchet distance takes into account the course of the curves. Decide if $\delta_F(P, Q) \leq \varepsilon$ in $O(mn)$.

Curve simplification

Find $P'$ an $\varepsilon$-simplification of $P$

$\delta_F(P, P') \leq \varepsilon$

minimize the number of vertices of $P'$

Optimal algorithm in $O(n^3)$.

A nice local property [2]

Let $P = \{p_1, p_2, \ldots, p_k\}$ be a polygonal curve.

For $1 \leq i \leq l \leq j \leq k$, $\text{error}(i, j) \leq \sqrt{2} \text{error}(i, r)$.

Algorithm to compute an $\varepsilon$-simplification with at most the number of vertices of an optimal $\frac{\varepsilon}{\sqrt{2}}$-simplification in $O(n \log(n))$

Guaranteed Algorithm with approximated distance

The Fréchet error of a shortcut $p_i p_j$ satisfies [1]

$$\max\{w(i,j), b(i,j)\} \leq \frac{1}{\sqrt{2}} \text{error}(i, j) \leq 2\sqrt{2} \max\{w(i,j), b(i,j)\}$$

$\implies$ Computes an $\varepsilon$-simplification with at most the number of vertices of an optimal $\frac{\varepsilon}{\sqrt{2}}$-simplification

$\implies$ Complexity ? Efficient update?

Approximated distance update

Digital curve : only 8 elementary shifts

$\downarrow$

$\Rightarrow$ occulter = locally furthest point in direction $\downarrow$

$\Rightarrow$ origin of the longest backpath = occulter

Keypoint : knowing when the curve goes forward or backward in a given direction

An elementary shift

always goes forward
always goes backward

for all the directions of an octant

For digital curves, the number of active occulters per octant is bounded by $\varepsilon$.

Overall complexity $O(n \log(n))$

Results

$\varepsilon = 8$

Runtime results for noisy synthetic shapes.

Runtime results for noisy flowers with different values for $\varepsilon$.

Approx. Fréchet distance

60 vertices

Width criterion

56 vertices

References

