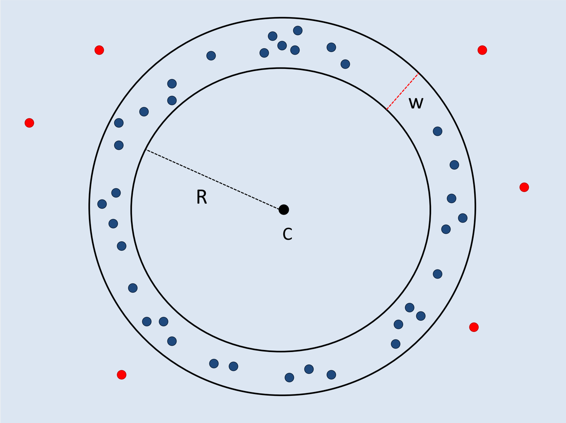
Optimal consensus set for annulus fitting

Rita Zrour, Gaëlle Largeteau-Skapin, Eric Andres

{rzrour,glargeteau,andres}@sic.univ-poitiers.fr
Department SIC, Laboratory XLIM, UMR CNRS 6172
University of Poitiers
BP 30179, 86962 Futuroscope Chasseneuil Cedex, France

Contribution



A method for fitting fixed width annulus to a given set of points in a 2D images in the presence of outliers

- examines all possible consensus sets,
- guarantees the optimal and exact solution(s),
- has a time complexity $O(N^4)$ with N the number of points.

Annulus fitting

An annulus A of width ω and radius R centered at $C(C_x, C_y)$, is defined by the set of points in R^2 satisfying two inequalities:

$$S = \left\{(P_x, P_y) \in \mathbb{R}^2 : R^2 \leq (P_x - C_x)^2 + (P_y - C_y)^2 \leq (R+\omega)^2\right\} \quad (1)$$

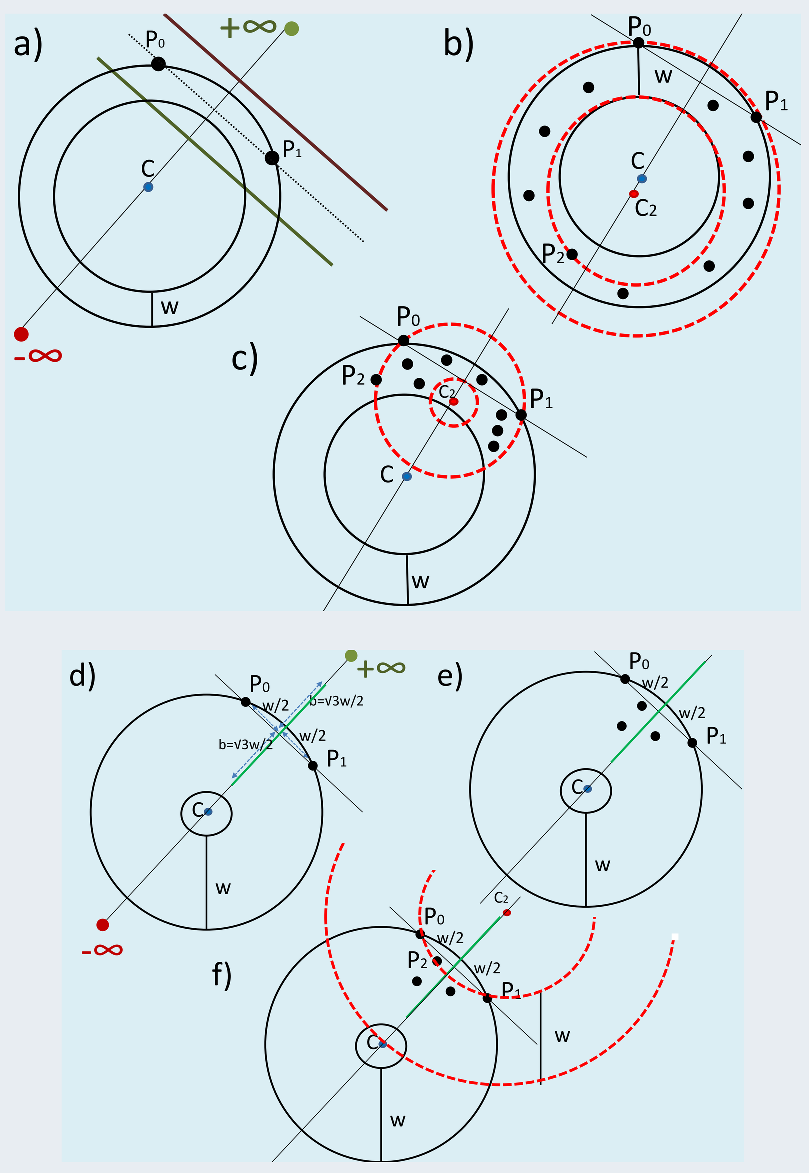
where $C(C_x, C_y) \in \mathbb{R}^2$ and R, $\omega \in \mathbb{R}_+$.
Given a finite set $S = \left\{(P_x, P_y) \in \mathbb{R}^2\right\}$ of n points we would like to find an annulus A of width ω such that it contains the maximum number of points in S

Annular characterizations

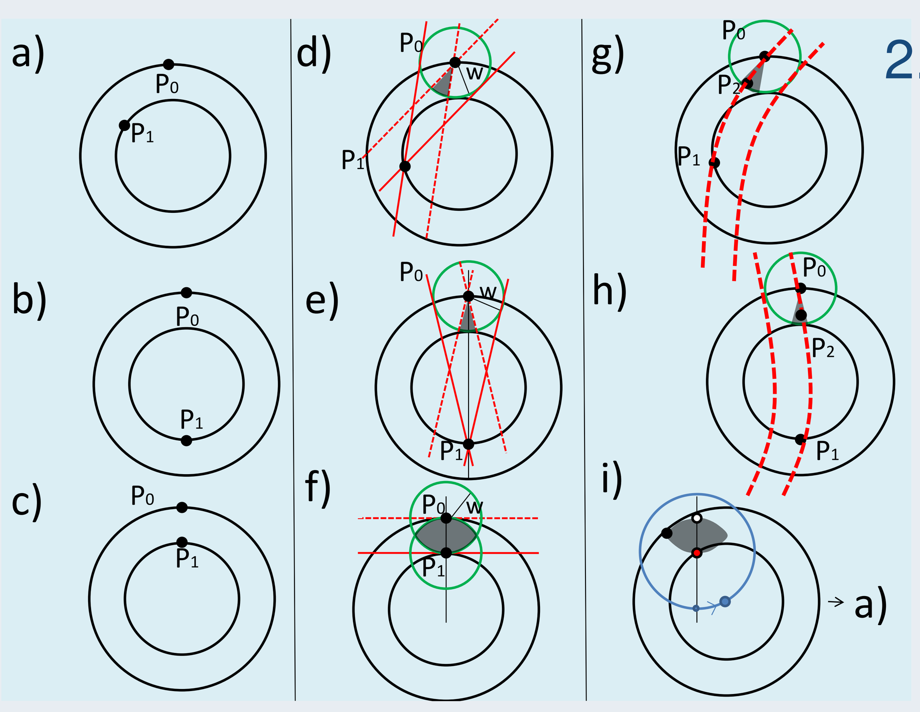
Theorem Given a width ω, and given an annulus A covering a set of points S, there exists at least another annulus A' of same width, that covers S and passes through at least 3 points of S.

Proof.

- first step : radius decreasing until reaching a point P_0
- second step : rotation centered on P_0 until reaching a second point P_1
- third step : **Two configurations** can appear.



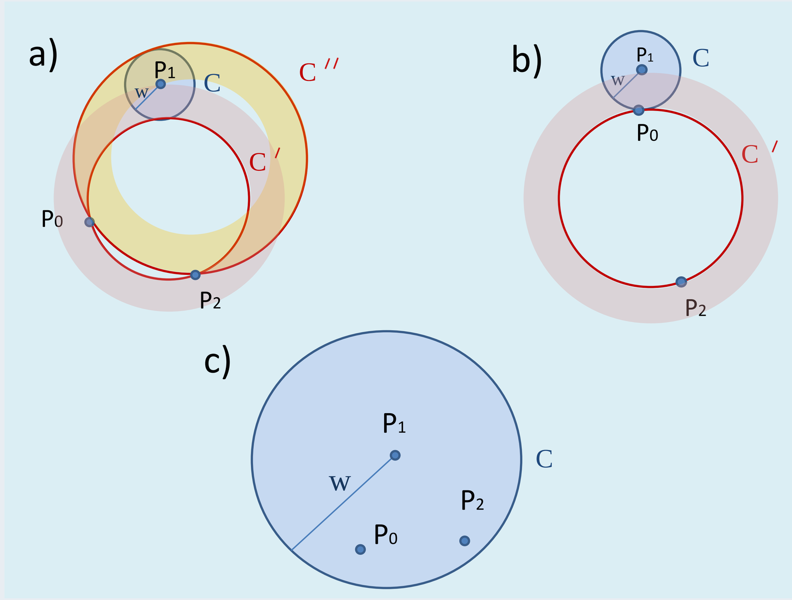
1. **both points are on one border**: depending on $d(P_0P_1)$ the centered is moved in order to reach a third point P_2. Fig. a, b and c, show the case - $d(P_0P_1) \geq 2\omega$. The center C colored blue must be moved along the line $-\infty$ in b) and $+\infty$ in c) in order to reach a third point P_2. The new center is C_2. Fig. d, e, f show the case - $d(P_0P_1) < 2\omega$. The configuration must be changed by choosing a point P_2 closest to the external border and the new annulus is the one colored in red.



2. **the two points are each on a different border** (Fig. a, b, c): modifying the radius allows to reach almost all the initial set. However, there exists an area that is not reached (in dark in Fig. d, e, f). If the points are all in this area, we have to change our strategy (see Fig. g, h, i).

Building an annulus of width ω from three points

Theorem There are at most 8 annuli of a given width ω passing through 3 given points P_1, P_2 and P_3 of S.



Proof.

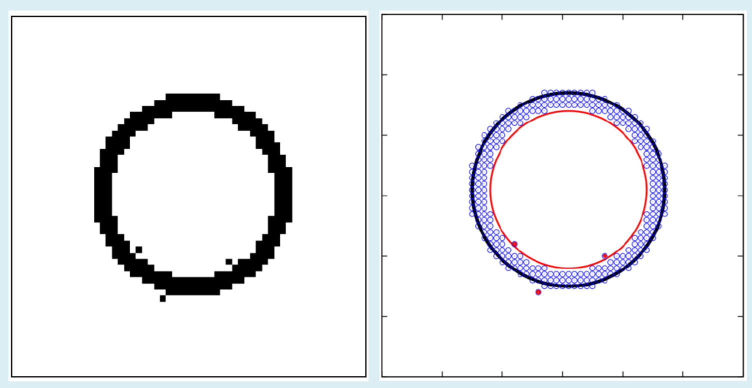
- if the 3 points are on the same circle: radius $> \omega$ then 2 annular are built, radius $< \omega$, only one,
- if 2 of the 3 points are on one border and the third one on the other: for each configuration we can buid at most two annuli (Fig. a, b, c). Fig. a $\Rightarrow$ 2 solutions, Fig. b $\Rightarrow$ 1 solution, Fig. c $\Rightarrow$ 0 solution.

Method : test all configurations of 3 points and count, for each of the possible 8 configurations, the points inside the annuli. This yields a $O(N^4)$ complexity, with N the number of points.

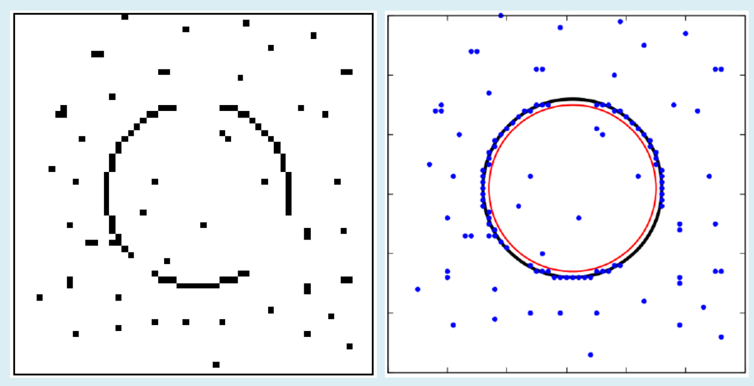
Experiments

2D noisy digital Andres circles and an Andres arc.

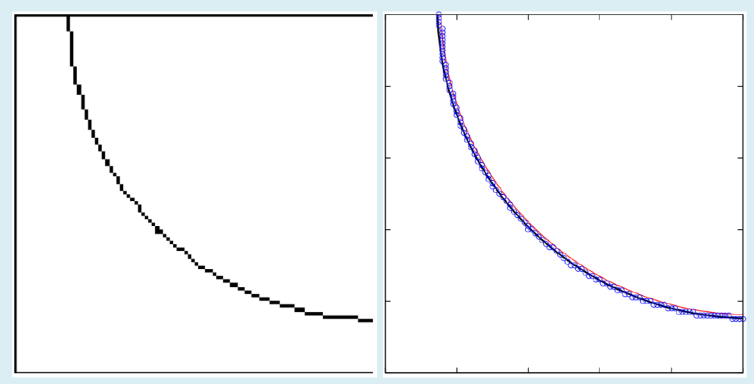
Data		Results		
Number of points	Thickness w	Center position	R	Opt. consensus set size
289	3	(31,31)	13	286
121	1	(101.581,102.226)	86	118
119	1	(31,31)	14	65
309	1	(31,31) (49,49)	19	114



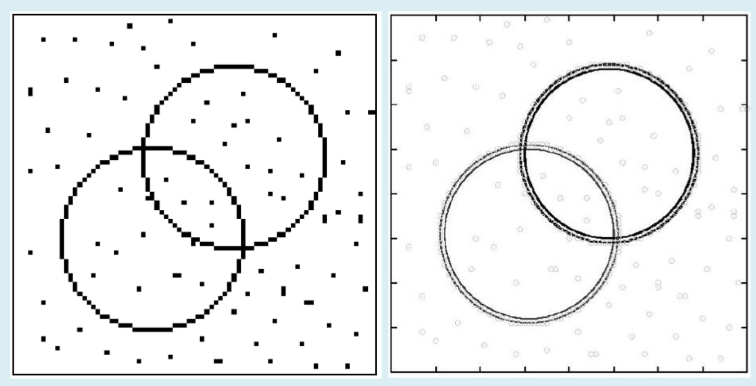
Andres circle of width 3



Andres circle of width 1



Andres arc of width 1



Two optimal consensus sets can be fitted

Conclusion

- fitting annulus to a set of points while fixing the width of the annulus,
- approach costly in terms of computation time $O(N^4)$ complexity,
- guarantees optimal and exhaustive results,
- fit an annulus with the least amount of outliers.

Perspectives

- improving the complexity ($O(N^3 logN)$),
- fitting of 3D sphere and extension to nD.

Acknowledgements

The authors express their thanks to the NavII FEDER Project, co-funded by Poitou-Charentes region.