Completions and simplicial complexes

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Completions

- Completions are inductive properties which may be expressed in a declarative way and which may be combined.
- We show that completions may be used for describing structures or transformations which appear in combinatorial topology.

All completions $\langle \kappa \rangle$ have the following form:

$\rightarrow$ If $\mathbf{F} \subseteq \mathcal{K}$, then $\mathbf{G} \subseteq \mathcal{K}$ whenever $(\mathbf{F}, \mathbf{G}) \in \kappa$. $\langle \kappa \rangle$

- The symbol κ stands for a binary relation over $2^{\mathbf{S}}$ and $2^{\mathbf{S}}$.
- $\mathbf{F}$ must be finite.

Theorem Let $\mathbf{X} \subseteq \mathbf{S}$. There exists a unique minimal collection which contains $\mathbf{X}$ and which satisfies $\langle \kappa \rangle$.
We write $\langle \mathbf{X}, \kappa \rangle$ for this collection.

The Cup/Cap completions

We introduce the notion of a dendrite for defining a remarkable collection made of acyclic complexes.

We define the two completions $\langle \text{Cup} \rangle$ and $\langle \text{Cap} \rangle$:
–> If S, $T \in \mathcal{K}$, then $S \cup T \in \mathcal{K}$ whenever $S \cap T \in \mathcal{K}$. $\langle \text{Cup} \rangle$
–> If S, $T \in \mathcal{K}$, then $S \cap T \in \mathcal{K}$ whenever $S \cup T \in \mathcal{K}$. $\langle \text{Cap} \rangle$
We set $\mathbb{D} = \langle \mathbb{C}, \text{Cup}, \text{Cap} \rangle$. Each element of $\mathbb{D}$ is a *dendrite*.
The symbol $\mathbb{C}$ stands for the collection of all cells (points, segments, triangles, tetrahedra...).

Theorem.
A simplicial complex is a dendrite if and only if it is contractible.

Thank you for your attention.