

# Circular Arc Reconstruction of Digital Contours with Chosen Hausdorff Error

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# 1. Context

Discrete contour representation :

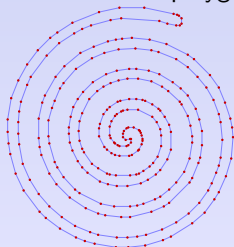
- Many approaches exist for the problem of discrete curve representation with polygonal approximation (see for instance the recent work of [Feschet 2010] or [Sivignon 2011]).
- Address the problem of contour approximation by circular arcs.
- More compact representation for numerous shapes.

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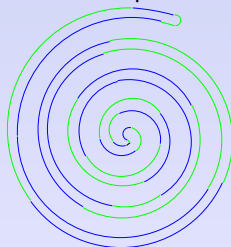
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Curvature based polygon



$\delta_H = 2.018$   
288 segments

Circular arc representation

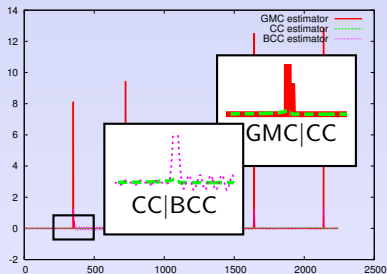
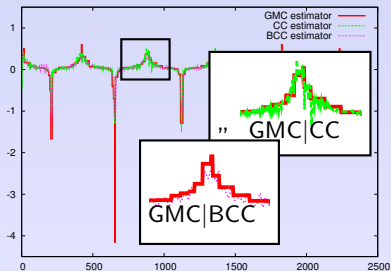


$\delta_H = 2.011$   
27 arcs, 1 segment

## 2. Main idea

### Reconstruction based on curvature estimators :

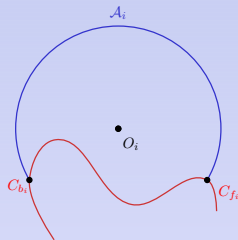
- Exploit recent and robust to noise curvature estimators GMC [Kerautret and Lachaud 2009] or BCC [Malgouyres *et al.* 2008].
- Based on a simple split/merged based algorithm.
- Approximation of the Hausdorff distance  $\delta_H$  between arc/source contour.



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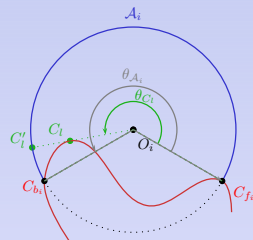


$$\delta_H(\mathcal{A}_i, \mathcal{C}_i) = \max\left\{\max_{b \in \mathcal{C}_i} \left\{\min_{a \in \mathcal{A}_i} d(a, b)\right\}, \max_{a \in \mathcal{A}_i} \left\{\min_{b \in \mathcal{C}_i} d(a, b)\right\}\right\}$$

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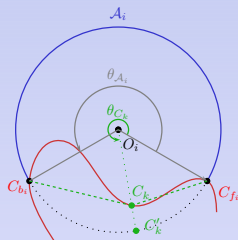


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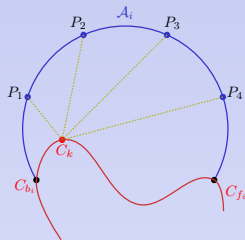


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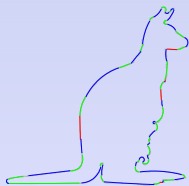


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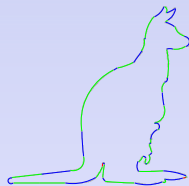
### 3. Results and comparisons

Comparisons by using other curvature estimator BCC [Malgouyres *et al.* 2008], and with recent method NASR [T.P. Nguyen 2010],

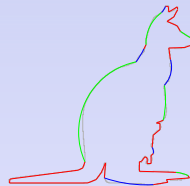
scale=2



GMC :  $\bar{A} = 53$ ,  $\bar{S} = 5$   
184 ms.  $\delta_H = 2.06539$



BCC  $\bar{A} = 53$ ,  $\bar{S} = 2$   
961 ms.  $\delta_H = \mathbf{1.99648}$

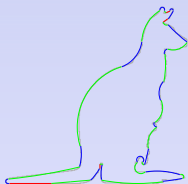


NASR  $\bar{A} = 10$ ,  $\bar{S} = 40$   
115 ms.  $\delta_H = 8.81655$

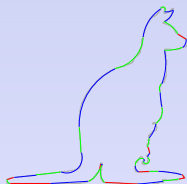
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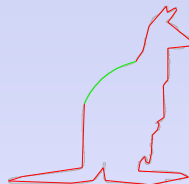
scale=4



GMC :  $\bar{A} = 31$ ,  $\bar{S} = 4$   
261 ms.  $\delta_H = \mathbf{3.94931}$



BCC  $\bar{A} = 33$ ,  $\bar{S} = 9$   
2990 ms.  $\delta_H = 6.11619$

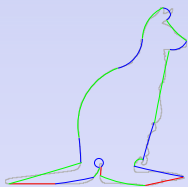


NASR  $\bar{A} = 1$ ,  $\bar{S} = 34$   
138 ms.  $\delta_H = 9.84886$

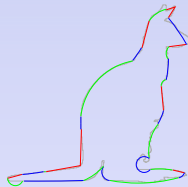
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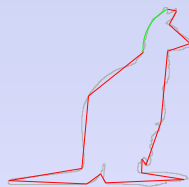
scale=10



GMC :  $\bar{A} = 17$ ,  $\bar{S} = 3$   
464 ms.  $\delta_H = \mathbf{10.2849}$



BCC  $\bar{A} = 20$ ,  $\bar{S} = 9$   
15596 ms.  $\delta_H = 10.5289$

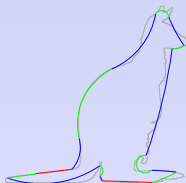


NASR  $\bar{A} = 1$ ,  $\bar{S} = 16$   
171 ms.  $\delta_H = 19.6977$

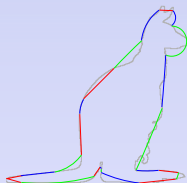
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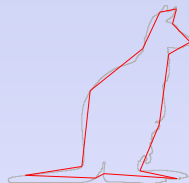
scale=15



GMC :  $\bar{A} = 14$ ,  $\bar{S} = 2$   
619 ms.  $\delta_H = \mathbf{17.2402}$



BCC  $\bar{A} = 15$ ,  $\bar{S} = 8$   
33420 ms.  $\delta_H = 17.7125$



NASR  $\bar{A} = 0$ ,  $\bar{S} = 14$   
190 ms.  $\delta_H = 32.8938$

### 3. More details ...

- Others comparisons with Visual Curvature polygonalisation method [Liu *et al.* 2008].
- More results available on the demonstration session.



[Sivignon 2011] I. Sivignon : A Near-Linear Time Guaranteed Algorithm for Digital Curve Simplification under the Fréchet Distance. In : Proc of DGCI 2011. pp. 333–345 (2011)



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[Liu *et al.*2008] Liu, H., Latecki, L.J., Liu, W. : A unified curvature definition for regular, polygonal, and digital planar curves. Int. J. Comput. Vision 80(1), 104–124 (2008)



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[T.P. Nguyen a2010] Nguyen, T.P. : Etude des courbes discrètes : applications en analyse d'images. Ph.D. thesis, Nancy University - LORIA (2010), (in french)