

Circular Arc Reconstruction of Digital Contours with Chosen Hausdorff Error

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1. Context

Discrete contour representation :

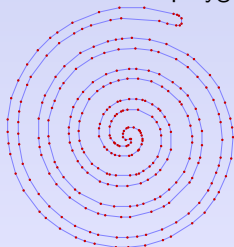
- Many approaches exist for the problem of discrete curve representation with polygonal approximation (see for instance the recent work of [Feschet 2010] or [Sivignon 2011]).
- Address the problem of contour approximation by circular arcs.
- More compact representation for numerous shapes.

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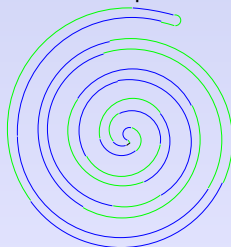
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Curvature based polygon



$\delta_H = 2.018$
288 segments

Circular arc representation

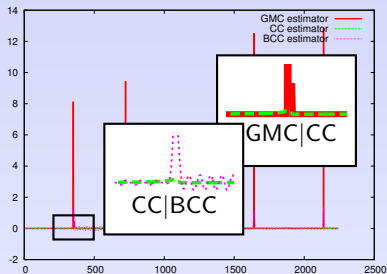
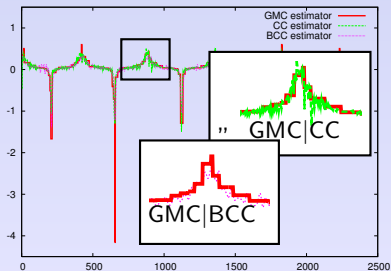


$\delta_H = 2.011$
27 arcs, 1 segment

2. Main idea

Reconstruction based on curvature estimators :

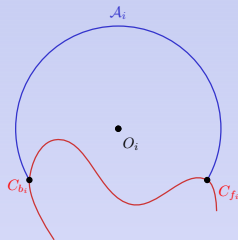
- Exploit recent and robust to noise curvature estimators GMC [Kerautret and Lachaud 2009] or BCC [Malgouyres *et al.* 2008].
- Based on a simple split/merged based algorithm.
- Approximation of the Hausdorff distance δ_H between arc/source contour.



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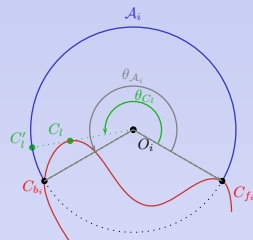


$$\delta_H(\mathcal{A}_i, \mathcal{C}_i) = \max\left\{\max_{b \in \mathcal{C}_i} \left\{\min_{a \in \mathcal{A}_i} d(a, b)\right\}, \max_{a \in \mathcal{A}_i} \left\{\min_{b \in \mathcal{C}_i} d(a, b)\right\}\right\}$$

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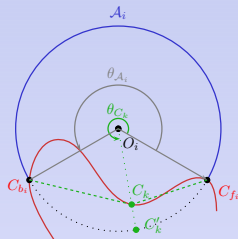


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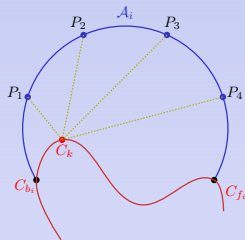


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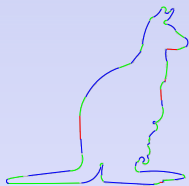


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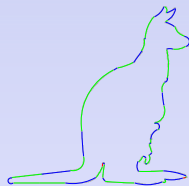
3. Results and comparisons

Comparisons by using other curvature estimator BCC [Malgouyres *et al.* 2008], and with recent method NASR [T.P. Nguyen 2010],

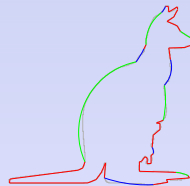
scale=2



GMC : $\bar{A} = 53$, $\bar{S} = 5$
184 ms. $\delta_H = 2.06539$



BCC $\bar{A} = 53$, $\bar{S} = 2$
961 ms. $\delta_H = \mathbf{1.99648}$

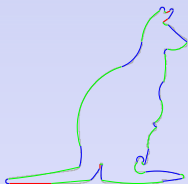


NASR $\bar{A} = 10$, $\bar{S} = 40$
115 ms. $\delta_H = 8.81655$

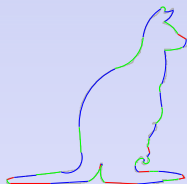
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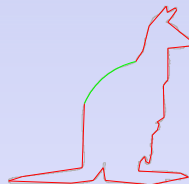
scale=4



GMC : $\bar{A} = 31$, $\bar{S} = 4$
261 ms. $\delta_H = \mathbf{3.94931}$



BCC $\bar{A} = 33$, $\bar{S} = 9$
2990 ms. $\delta_H = 6.11619$

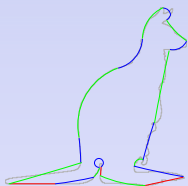


NASR $\bar{A} = 1$, $\bar{S} = 34$
138 ms. $\delta_H = 9.84886$

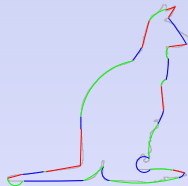
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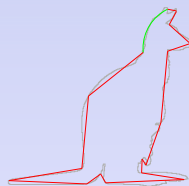
scale=10



GMC : $\bar{A} = 17$, $\bar{S} = 3$
464 ms. $\delta_H = \mathbf{10.2849}$



BCC $\bar{A} = 20$, $\bar{S} = 9$
15596 ms. $\delta_H = 10.5289$

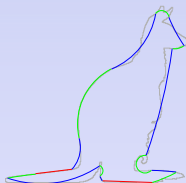


NASR $\bar{A} = 1$, $\bar{S} = 16$
171 ms. $\delta_H = 19.6977$

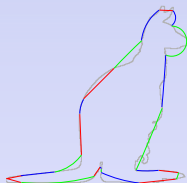
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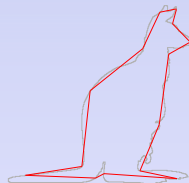
scale=15



GMC : $\bar{A} = 14$, $\bar{S} = 2$
619 ms. $\delta_H = \mathbf{17.2402}$



BCC $\bar{A} = 15$, $\bar{S} = 8$
33420 ms. $\delta_H = 17.7125$



NASR $\bar{A} = 0$, $\bar{S} = 14$
190 ms. $\delta_H = 32.8938$

3. More details ...

- Others comparisons with Visual Curvature polygonalisation method [Liu *et al.* 2008].
- More results available on the demonstration session.



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[Liu *et al.*2008] Liu, H., Latecki, L.J., Liu, W. : A unified curvature definition for regular, polygonal, and digital planar curves. Int. J. Comput. Vision 80(1), 104–124 (2008)



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[T.P. Nguyen a2010] Nguyen, T.P. : Etude des courbes discrètes : applications en analyse d'images. Ph.D. thesis, Nancy University - LORIA (2010), (in french)